## NOTATION

$R_2$, radius of outer sphere; $R_1$, radius of inner sphere; v, velocity of solution; $\nu$, kinematic viscosity; g, acceleration; $c_0$, ion concentration; D, diffusion coefficient; $\rho$, solution density; n, unit normal vector; $\beta = \rho^{-1}\partial\rho/\partial c$.

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# TEMPERATURE FIELD OF A PLATE WITH INTERNAL TEMPERATURE-DEPENDENT HEAT SOURCE


N. I. Gamayunov and A. V. Klinger



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The problem of asymmetric heating of a plate is considered in a medium with variable temperature in the presence of an internal heat source, the power of which is dependent on temperature and time.


In many engineering problems related to calculation of heat-transfer processes, it becomes necessary to analyze the effect on the temperature field of the body under study of internal heat sources, usually those produced by exothermal chemical reactions. In calculations the power of such heat sources is usually taken as constant, or its dependence on time and coordinate is specified in the form of certain known functions which make possible use of existing solutions of the thermal conductivity equation for calculation of the temperature field [1]. However, in the majority of real processes, the internal heat source power is significantly dependent on temperature. Thus, in hardening of a number of structural materials, hydration of various cement substances takes place, accompanied by heat liberation. With increase in temperature the intensity of the hydration reaction increases, so that heat liberation also increases. With the passage of time the initial reagent concentrations decrease, leading to a slowage of the reaction and heat liberation. A detailed analysis of heat liberation in hardening concrete was performed in [2]. The analysis reveals that the temperature–time dependence of the quantity of heat $Q_e$, liberated upon hardening 1 kg of cement, can be written in the form $Q_e = f^*(\tau)t$. The power of the internal heat source is proportional to the derivative

$$\frac{\partial Q_e}{\partial \tau} = \frac{\partial f^*(\tau)}{\partial \tau} t + f^*(\tau)\frac{\partial t}{\partial \tau}.$$

With consideration of this fact, the differential equation for heat transfer for the plate has the form

$$[1 - f(\tau)]\frac{\partial t}{\partial \tau} = a\frac{\partial^2 t}{\partial x^2} + \frac{\partial f(\tau)}{\partial \tau} t, \tag{1}$$

where $f(\tau) = \frac{M}{c\gamma} f^*(\tau)$; M is the quantity of cement in 1 $m^3$ of the concrete mixture. In [2] solutions of Eq. (1) were obtained for a number of special cases. Below a more general solution will be attempted.

Locating the origin of the coordinate system at the center of the plate, we write the initial boundary conditions of the problem in the form

$$t(x, 0) = t_0, \tag{2}$$

$$\left[(-1)^i \lambda \frac{\partial t}{\partial x} + \alpha_i t\right]\Big|_{x=X_i} = \alpha_i t_m(\tau), \tag{3}$$


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where $X_i = (-1)^i R$ is the coordinate of the i-th surface of the plate ($i = 1, 2$); $t_m(\tau)$ is the time dependence of the medium's temperature.

To solve Eq. (1) we introduce a finite integral transform over coordinate of the form

$$G = \frac{1}{K} \int_{-R}^{R} g(x) N(x)\, dx, \tag{4}$$

where G is a transform of the function g(x); N(x), an eigenfunction of the Sturm–Liouville problem [3]; and K, a normalizing factor.

Solving the Sturm–Liouville equation for the plate

$$N'' + \rho^2 N = 0$$

with homogeneous boundary conditions

$$[(-1)^i N'\lambda + \alpha_i N]|_{x=X_i} = 0 \ (i = 1,\ 2),$$

we find the eigenfunction in the form

$$N_n(x) = \sum_{i=1}^{2} \alpha_i (\lambda\rho_n \cos \rho_n \zeta_i + \alpha_{3-i} \sin \rho_n \zeta_i),$$

where $\zeta_i = R + (-1)^i x$; $\rho_n$ is a root of the characteristic equation

$$\lambda\rho (\alpha_1 + \alpha_2) \cos 2\rho R = (\lambda^2\rho^2 - \alpha_1\alpha_2) \sin 2\rho R.$$

Integrating the square of the eigenfrequency over coordinate from one surface of the plate to the other, we determine the normalizing factor

$$K_n = \frac{1}{D_n} [\lambda\rho_n (\alpha_1 + \alpha_2) \sin 2\mu_n + 2\alpha_1\alpha_2 (1 - \cos 2\mu_n)],$$

where

$$\mu_n = \rho_n R;\ D_n = (\rho_n \sin 2\mu_n) \Big/ \left(\lambda\rho_n\mu_n (\alpha_1 + \alpha_2) + \frac{1}{2} (\lambda^2\rho_n^2 + \alpha_1\alpha_2) \sin^2 2\mu_n\right).$$

Applying the integral transform (4) to Eqs. (1), (2), to find the transform of the temperature $T_n(\tau)$ we obtain the ordinary differential equation

$$\frac{dT_n}{d\tau} + \frac{T_n}{1 - f(\tau)} \left[a\rho_n^2 - \frac{df(\tau)}{d\tau}\right] = \frac{a\rho_n D_n}{1 - f(\tau)} t_m(\tau)$$

with initial condition

$$T_n(0) = \frac{D_n}{\rho_n} t_0,$$

the solution of which has the form

$$T_n(\tau) = \frac{D_n \psi_n(\tau)}{\rho_n [1 - f(\tau)]} \{t_0 [1 - f(0)] + H_n(\tau)\},$$

where

$$\psi_n(\tau) = \exp\left[- a\rho_n^2 \int_0^{\tau} \frac{d\vartheta}{1 - f(\vartheta)}\right];\ H_n(\tau) = a\rho_n^2 \int_0^{\tau} \frac{t_c(\theta)}{\psi_n(\theta)}\, d\theta.$$

Using the reduction formula of [3], we write the final solution in the form of a series

$$t(x,\ \tau) = \sum_{n=1}^{\infty} T_n(\tau) N_n(x). \tag{5}$$

The form of the function $f(\tau)$ depends on the concrete conditions under which the hydration reaction occurs. In [2], in particular, the function

$$f(\tau) = m\,[1 - \exp(-h\tau)] \tag{6}$$

was proposed, where m and h are constants. In this case the function $T_n(\tau)$ takes on the form

$$T_n(\tau) = \frac{D_n \exp(h\tau)}{\rho_n\,[\omega(\tau)]^{p_n+1}} \left\{t_0 + a\rho_n^2 \int_0^{\tau} t_m(\theta)\,[\omega(\theta)]^{p_n} d\theta\right\},$$

where

$$\omega(\tau) = m + (1 - m)\exp(h\tau);\quad p_n = \frac{a\rho_n^2}{h(1-m)}.$$

In the first 10-12 h of the hydration process the function $f(\tau)$ can be approximated by a linear expression [2]

$$f(\tau) = k\tau. \tag{7}$$

Thus, for the initial stage of the process, we obtain

$$T_n(\tau) = \frac{D_n}{\rho_n}(1 - k\tau)^{q_n - 1}\left[t_0 + a\rho_n^2 \int_0^{\tau} t_m(\theta)(1 - k\theta)^{-q_n} d\theta\right], \tag{8}$$

where $q_n = a\rho_n^2/k$ .

We will illustrate the practical use of the results obtained with the example of high-temperature thermal processing of a part made of light concrete. For this process, short-term heating regimes are characteristic, permitting use of Eq. (7) in the calculations.

In the first stage of the thermal processing ($\tau \leq \tau_1$) the temperature of the medium changes linearly with time $t_{m1}(\tau) = t_0 + b\tau$. Substituting this expression in Eq. (8), we obtain

$$T_n(\tau)|_{\tau\leq\tau_1} = P_n \Phi_n(\tau), \tag{9}$$

where

$$\Phi_n(\tau) = t_{m1}(\tau) - s_n(1 - k\tau) + \left(s_n - \frac{t_0}{q_n}\right)(1 - k\tau)^{q_n - 1};$$

$$P_n = \frac{D_n q_n}{\rho_n(q_n - 1)};\quad s_n = \frac{b}{k(q_n - 2)}.$$

In the second stage of the process ($\tau > \tau_1$) the temperature of the medium $t_{m2}$ remains constant, equal to $t_0 + b\tau_1$. In this case Eq. (8) takes on the form

$$T_n(\tau)|_{\tau>\tau_1} = P_n\left\{t_{m1} + [\Phi_n(\tau_1) - t_{m1}]\left(\frac{1 - k\tau}{1 - k\tau_1}\right)^{q_n - 1}\right\}. \tag{10}$$

If in Eqs. (9), (10) we set k = 0 (absence of an internal heat source), after substitution in Eq. (5) we obtain solutions identical to those of [4].

Plate temperature fields were calculated by a computer with Eq. (5) with the aid of Eqs. (9), (10). The thermophysical properties of the material were determined from experimental data by the method described in [5], and were taken as follows: $a = 2.5 \cdot 10^{-3}$ m²/h; $\alpha_1/\lambda = 20$ m$^{-1}$; $\alpha_2/\lambda = 30$ m$^{-1}$. The value of the coefficient $k = 0.04$ h$^{-1}$ was chosen from the recommendations of [2]. The time dependence of the medium's temperature is shown in Fig. 1, which shows results of calculations with and without consideration of the internal heat source.

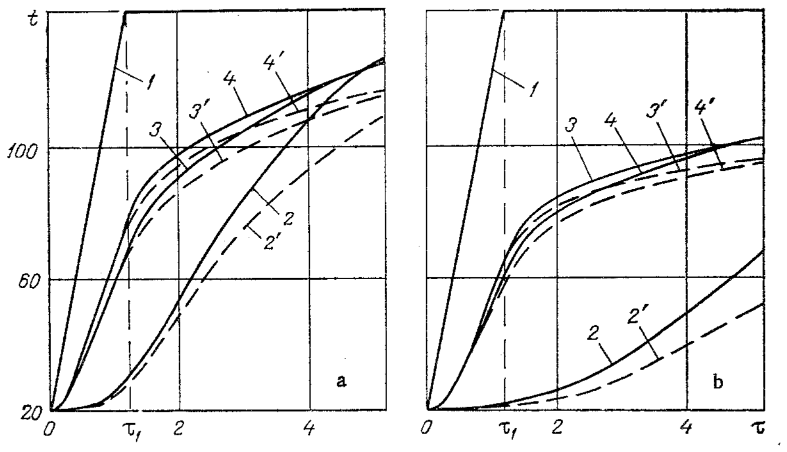


Fig. 1. Temperature t (°C) of medium (1), center (2, 2'), and surfaces (3, 3', 4, 4') of plate vs time $\tau$ (h) with (2, 3, 4) and without (2', 3', 4') consideration of internal heat source: a) R = 0.1 m; b) R = 0.2 m.

It is evident from comparison of Fig. 1a and b that the heat of the exothermal reaction has a larger effect on the temperature of the central layers of the plate, while with one and the same heating regime for a plate of larger thickness the change of the temperature field under the action of the internal heat source is more significant.

The solutions obtained can be used for theoretical analysis of the effect of heating regime parameters on internal heat-transfer processes in bodies in which exothermal reactions occur, in particular, in hardening concrete. The method proposed permits selection of optimal thermal processing of concrete slabs.

## NOTATION

x, coordinate; $\tau$, time; t, temperature; $\lambda$ and $a = \lambda/c\gamma$, thermal conductivity and diffusivity coefficients; c, specific heat; $\gamma$, density; $\alpha_i$, heat liberation coefficient at i-th surface of plate (i = 1, 2); R, geometrical dimension (half of plate thickness).

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